PES INSTITUTE OF TECHNOLOGY BANGALORE SOUTH CAMPUS

QUESTION BANK

DIGITAL SIGNAL PROCESSING (17EC52)

Module 1

1. The even samples of the $11-\text{point}$ DFT of a $\text{length}-11$ real sequence are given by $X(0)=2, X(2)=-1-3j, X(4)=1+4j, X(6)=9+3j, X(8)=5, X(10)=2+j2$. Determine the missing odd samples of the DFT.
2. Compute the $4-\text{point}$ DFT of the sequence $x(n)=\cos\left(\frac{\pi}{4}n\right)+\sin(\frac{\pi}{4}n)$.
3. Find the $6-\text{point}$ DFT of the sequence $x(n)=\{1,1,2,2,3,3\}$. Draw magnitude and phase spectra.
4. Determine the $N-\text{point}$ DFT of $x(n)=\frac{1}{2}+cos^2(\frac{2\pi n}{N})$ where $n=0,1,2,\dots N-1$.
5. Given $x(n)=\{1,1,1\}$, obtain the $5-\text{point}$ DFT $X(k)$.
6. Obtain the $10-\text{point}$ DFT of the sequence $x(n)=\delta(n)+2\delta(n-5)$.
7. Find the $N-\text{point}$ DFT of $x(n)=\cos(\frac{2\pi}{N}k_0 n)$.
8. Find the $5-\text{point}$ DFT of $x(n)=\{1,1,1\}$.
9. Find the IDFT of the sequence $X(k)=\{5,0,1-j,0,1,0,1+j,0\}$.

1. Define Z Transform for a discrete time signal x(n). Explain the significance of ROC in Z Transform.

2. Derive the expression for DFT from DTFT expression.

3. Show that DFT and IDFT form a consistent Discrete Fourier Transform pairs.

4. Establish the relation between DFT and ZT.

5. Establish the relation between DFT and DFS.

6. What are twiddle factors in DFT?

7. Explain the methods for computation of DFT in linear filtering of long duration sequences with appropriate diagrams.

8. Compute the DFT of the sequence x(n) = {0, 1, 2, 3}.

9. Calculate the 8 point DFT of the sequence x(n) = {1, 1, 1, 1}.

10. Compute the DFT of the following standard signals: (a) x(n) = δ(n); (b) x(n) = a^n for $0 \le n \le N-1$;

Module 2

11. State and prove the following properties of DFT: (a) Periodicity; (b) Linearity; (c) Time shifting property; (d) Time reversal of a sequence; (e) Complex conjugate property; (f) Multiplication of 2 DFT's – Circular Convolution property; (g) Circular Correlation property; (h) Multiplication of two sequences; (i) Parseval's Theorem; (j) Circular Time Shift property; (k) Circular Frequency Shift property;

12. State the periodic convolution of two discrete sequences.

13. Distinguish between linear convolution and circular convolution of two sequences.

14. Give the steps to get the result of linear convolution from the method of circular convolution.

15. Explain the meaning of 'frequency resolution' of the DFT.

16. Convolve the following sequences using (a) overlap – add method; (b) overlap – save method; for x(n) = {1, -1, 2, 1, 2, -1, 1, 3, 1} and h(n) = {1, 2, 1}.

17. Determine the DFT of the sequence $h(n)=\begin{Bmatrix} \frac{1}{3}, 0 \le n \le 2 \\ 0, \text{otherwise} \end{Bmatrix}$.

18. Compute the inverse DFT for the sequence $X(k) = e^{-k}$ for k = 16.

19. For the 8 sample sequence x(n) = {1, 2, 3, 5, 5, 3, 2, 1}, the first 5 DFT coefficients are {22, -7.5355 – j 3.1213, 1 + j, -0.4645 – j 1.123, 0}. Determine the remaining 3 DFT coefficients.

20. The six samples of the $11-\text{point}$ DFT of a $\text{length}-11$ real sequence are given by $X(0)=12, X(2)=-3.2-j2, X(3)=5.3-j4.1, X(5)=6.5+j9, X(7)=-4.1+j0.2,\ X(10)=-3.1+j5.2$. Determine the remaining samples of the DFT

Module 3

1. Give the number of complex additions and complex multiplications required for direct computation of N point DFT.

2. What is the need for Fast Fourier Transform (FFT) algorithm?

3. Compare the number of multiplications required to compute the DFT of a 64 point sequence using direct computation and that using FFT.

4. What is DIT-FFT algorithm?

5. What is DIF-FFT algorithm?

6. Which FFT algorithm procedure is the lowest possible level of DFT decomposition?

7. Give the computation efficiency of FFT over DFT.

8. Compute the FFT of the sequence $x(n)=n^2+1$, where, N = 8 using DIT algorithm.

9. Draw the flow graph of an 8 point DIF-FFT and explain.

10. Discuss the computational efficiency of radix 2 FFT algorithm.

11. Explain how you would use FFT algorithm to compute IDFT.

12. Develop the decimation-in-time and decimation-in-frequency FFT algorithms for decomposing DFT for N = 3.3.3 and obtain the corresponding signal flow graphs.

13. Determine the DFT of the following sequence using DIF-FFT algorithm: $x_1(n)$ = {1, 1, 1, 0, 0, 1, 1, 1}. Using the DFT of $x_1(n)$ find the DFT of the sequence $x_2(n)$ = {1, 1, 1, 1, 1, 0, 0, 1}.

14. What are the advantages of fast (sectioned) convolution?

Module 4

1. Explain the frequency response characteristics of Butterworth filters.

2. What are frequency transformations? Why is it required?

3. Given $|H_a(j\Omega)|^2 = \frac{1}{1+64\Omega^6}$, determine the analog filter system function $H_a(s)$.

4. Design an analog filter with maximally flat response in the passband and an acceptable attenuation of -2 dB at 20 rad/sec. The attenuation in stopband should be more than 10 dB beyond 30 rad/sec.

5. Design a low pass 1 rad/sec bandwidth Butterworth filter with the following characteristics: (i) Acceptable passband ripple of 2 dB; (ii) Cutoff radian frequency of 1 rad/sec; (iii) Stopband attenuation of 20 dB or greater beyond 1.3 rad/sec.

6. Using analog frequency transformation design an analog Butterworth filter with the following specifications: (i)Passband ripple: 1 dB for $0 \le \Omega \le 10$ rad/sec; (ii) Stopband ripple: - 60 dB for $\Omega \ge 50$ rad/sec.

7. Using analog frequency transformations design a highpass Butterworth filter of 4^{th} order for cutoff frequency of 50 Hz.

8. Using analog frequency transformations design a second order bandpass Butterworth filter with passband of 200 Hz to 300 Hz.

9. Determine the normalized low pass Butterworth analog poles for N = 10.

10. A difference equation describing a filter is given by y(n) – ¾ y(n-1) + 1/8 y(n-2) = x(n) + ½ x(n-1). Draw direct form I and direct form II structures.

11. Realize the following system function in parallel form

$$H(z) = \frac{1 - \frac{2}{3}z^{-1}}{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2}} \cdot \frac{1 + \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

12. Realize the following system function in cascade form $H(z) = \frac{(1 - z^{-1})^3}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{8}z^{-1}\right)}$

13. Obtain the parallel form realization of the following system function $H(z) = \frac{(3 + z^{-1})}{3\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$

14. An analog filter has the following system function. Convert this filter into a digital filter using impulse invariant technique: $H(s) = \frac{1}{(s + 0.1)^2 + 9}$

15. Use the backward difference for the derivative to convert the analog low pass filter with the following system function, using impulse invariant transformation H(s) = 1/(s+2).

16. Transform the analog filter with the transfer function shown below into a digital filter, using backward difference for the derivative: H(s) = 1/(s+2)(s+3).

17. Convert the analog filter to a digital filter whose system function is $H(s) = \frac{36}{(s + 0.1)^2 + 36}$. The digital filter should have a resonant frequency of $\omega_r = 0.2\pi$. Use bilinear transformation.

18. Convert the analog filter to a digital filter whose system function is $H(s) = \frac{1}{(s + 2)^2 (s + 1)}$ using bilinear transformation.

19. Design and realize a digital LPF using bilinear transformation to satisfy the following requirements: (i) monotonic passband and stopband; (ii) – 3 dB cut off frequency at 0.6π radians; (iii) magnitude down at 16 dB at 0.75π radians

20. Design a digital Butterworth filter to meet the following constraints using (i) Bilinear transformation; (ii) Impulse invariant method

$$0.8 \leq |H(e^{jw})| \leq 1, 0 \leq w \leq 0.2\pi$$

$$|H(e^{jw})| \leq 0.2, 0.26\pi \leq w \leq \pi$$

Module 5

1. A low pass filter has the desired response given by

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & 0 \le \omega \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le \omega \le \pi \end{cases}$$

Determine the filter coefficients $h(n)$ for $M = 7$ using frequency sampling technique.

2. The desired response of a low pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0, \quad \frac{3\pi}{4} \le |\omega| \le \pi \end{cases}$$

Determine $H(e^{j\omega})$ for $M = 7$ using a Hamming window.

3. Design a low pass filter with the following desired frequency response.

$$H_d(\omega) = H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & 0 \le |\omega| \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ and $h(n)$ if $w(n)$ is a rectangular window given by $w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

4. Design a low pass FIR filter with a desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & 0 \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi \end{cases}.$$

Use Hamming window with $M = 7$. Also obtain the frequency response.

5. A low pass filter is to be designed with the following frequency response:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega}, & 0 \le |\omega| \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ and $h(n)$ if $w(n)$ is a rectangular window given by $w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

Also find the frequency response $H(\omega)$ of the FIR filter.

6. Design a linear phase low pass FIR filter with 7 taps and cut off frequency of 0.3π rad, using the frequency sampling method.

7. Determine the filter coefficients $h(n)$ obtained by sampling $H_d(\omega)$ given by

$$H_d(\omega) = \begin{cases} e^{-j3\omega}, & 0 \le |\omega| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Also obtain the frequency response $H(\omega)$. Take $N = 7$.

1. An FIR filter is given by $y(n) = x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)$. Draw the direct form I and lattice structure.
2. Develop the lattice ladder structure for the filter with difference equation
$$y(n) + \frac{3}{4}y(n-1) + \frac{1}{4}y(n-2) = x(n) + 2x(n-1)$$
3. Realize the linear phase FIR filter having the following impulse response.
$h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4)$.
4. Obtain the direct form realization of linear phase FIR system given by $H(z) = 1 + \frac{2}{3}z^{-1} + \frac{15}{8}z^{-2} + \frac{2}{3}z^{-3} + z^{-4}$.