PES INSTITUTE OF TECHNOLOGY – BNAGALORE SOUTH CAMPUS
DEPARTMENT OF SCIENCE & HUMANITIES
SCHEME AND SOLUTION

CONTINUOUS INTERNAL EVALUATION TEST 2

Question Paper

Date : 02-04-2018		Marks: 60
Subject & Code : Engineering Physics – 17PHY22		Sec:A,B,C,D & E
Name of faculty :Dr. Muhammad Faisal, Dr. Mohana Lakshmi, Prof. Sneha L		Time :11:30a.m – 1:00p.m

Note: Answer FIVE full questions, choosing any ONE full question from each part
Physical Constants: $h = 6.63 \times 10^{-34}$ J.s, $k=1.38\times10^{-23}$ J/K, $C=3\times10^{8}$ m/s, $e= 1.6\times10^{-19}$ C, $m=9.1\times 10^{-31}$kg

			Marks
		PART 1	
1	a	Obtain an expression for the energy density of radiation in terms of Einstein's Coefficients.	8
	b	The ratio of the population of two energy states in a laser is 8.82×10^{-31}. If the temperature of the system is 57^0, what is the wavelength of the laser?	4
		OR	
2	a	With neat ray diagrams, explain the construction and reconstruction of Hologram.	6
	b	Describe the application of laser in welding, giving its advantages over conventional welding.	2
	c	A semiconductor laser emits a radiation of wavelength 1.24 µm. What is its band gap value in eV?	4
		PART 2	
3	a	Describe the construction of carbon dioxide laser and explain its working with the help of an energy level diagram. Give any two applications of a CO_2 laser.	12
		OR	
4	a	With neat diagrams, explain the salient features of the three different types of optical fibers with one application for each type.	12
		PART 3	
5	a	Obtain an expression for the numerical aperture of an optical fiber.	4
	b	Explain point to point communication system using optical fibers with the help of a block diagram.	4
	c	A signal with input power 200 mW loses 10% of its power after travelling a distance of 3000 m. Find the attenuation coefficient of the fiber.	4
		OR	
6	a	What is attenuation? Explain any two factors contributing to the fiber loss?	5
	b	Explain the terms (i) Acceptance angle (ii) Fractional index change (iii) V-number	3
	c	An optic glass fiber of refractive index 1.5 is to be clad with another glass to ensure total internal reflection that will contain light travelling within 5^0 of the fiber axis. What maximum index of refraction is allowed for the cladding?	4

PART 4			
7	**a**	What are the similarities and differences between the classical free electron theory and quantum free electron theory?	**6**
	b	Define the terms, (i) Mean Collision time (ii) Relaxation time	**2**
	c	Find the relaxation time for conduction electrons in aluminum metal.Given that for aluminum, the electron concentration=18 $x10^{22}/cm^3$ and resistivity = $2.7x10^{-8}\Omega m$.	**4**
OR			
8	**a**	Using the free electron model, derive an expression for electrical conductivity in metals.	**4**
	b	Discuss any two merits of quantum free electron theory.	**4**
	c	The Fermi level for a metal is 3.1 eV, calculate the energies for which the probability of occupancy at 300 k is 98% and 50%.	**4**
PART 5			
9	**a**	Define Fermi energy and Fermi velocity. Discuss the variation of Fermi factor with temperature and energy.	**2 + 6**
	b	For intrinsic gallium arsenide, the room temperature conductivity is $10^{-6}/\Omega m$. The electron and hole mobility's are respectively 0.85 m^2/Vs and 0.04 m^2/Vs. Calculate the intrinsic carrier concentration.	**4**
10	**a**	State law of mass action for a semiconductor. Derive an expression for electrical conductivity of an intrinsic semiconductor.	**2 + 6**
	b	Find the temperature at which there is 1 % probability that a state with energy 0.5 eV above Fermi level will be occupied.	**4**

SCHEME & SOLUTION

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Name of faculty: Dr. Muhammad Faisal, Dr. Mohana Lakshmi, Prof. Sneha **Time :** 11:30 a.m – 1:00 p.m

Q. no.		Scheme		Marks
PART 1	1	a	Expression for the energy density of radiation in terms of Einstein's Coefficients.	8
		b	Equation, Substitution & Answer	04
	2	a	Recording and reconstruction of Hologram	3+3
		b	Laser welding	02
		c	Equation, Substitution & Answer	04
PART 2	3	a	Construction and working of a carbon dioxide laser. Two applications	04+06+02
	4	a	Classification of different types of optical fibers	04*3
PART 3	5	a	Expression for numerical aperture of an optical fiber.	4
		b	Point to point optical fiber communication system	04
		c	Equation, Substitution & Answer	04
	6	a	Attenuation definition. Two factors contributing to attenuation	1+4
		b	Explanation of the terms (i) Acceptance angle (ii) Fractional index change (iii) V-number	3*1
		c	Equation, Substitution & Answer	04
PART 4	7	a	Similarities and differences between the classical free electron theory and quantum free electron theory	3+3
		b	Explanation of the terms (i) Mean Collision time (ii) Relaxation time	2*1
		c	Equation, Substitution & Answer	04
	8	a	Expression for electrical conductivity in metals.	04
		b	Two merits of quantum free electron theory.	2*2
		c	Equation, Substitution & Answer	04
PART 5	9	a	Definition of Fermi energy and Fermi velocity. Variation of Fermi factor with temperature	02+3*2
		b	Equation, Substitution & Answer	04
	10	a	Statement of law of mass action and expression for electrical conductivity of an ISC	2+6
		b	Equation, Substitution & Answer	04

SOLUTION
PART 1

1.a) Obtain an expression for the energy density of radiation in terms of Einstein's Coefficients.

Consider two energy states E_1 and E_2 of a system of atoms ($E_2>E_1$). Let there be N_1 atoms with energy E_1 and N_2 atoms with energy E_2, per unit volume of the system. N_1 and N_2 are called the number density of atoms in the states 1 and 2 respectively. Let radiations with a continuous spectrum of frequencies are incident upon the system. Let there be radiation of frequency $\gamma = (E_2-E_1)/h$, and let U_γ be the energy density of radiations of frequency γ. Then $U_\gamma d\gamma$ will be the energy density of radiations whose frequencies lie in the range γ and $\gamma + d\gamma$.

<u>Case of Induced Absorption:</u> In the case of induced absorption, an atom in the level E_1 can go to the level E_2 when it absorbs a radiation of frequency such that, $\gamma = (E_2-E_1)/h$. The number of such absorptions per unit time per unit volume, is called **rate of absorption.**

The rate of absorption depends upon,

a) the number density of lower energy state, i.e, N_1 and

b) the energy density i.e, U_γ

$\therefore$ Rate of absorption α N_1 U_γ

The Rate of stimulated absorption = $B_{12} N_1 U_\gamma$ ———— (1)

Where , B_{12} is the constant of proportionality called **Einstein coefficient of induced absorption.**

<u>Case of Spontaneous Emisssion:</u> In the case of spontaneous emission, an atom in the higher energy level E_2 undergoes transition to the lower energy level E_1 voluntarily by emitting a photon . Since it is a voluntary transition, it is independent of the energy density of any frequency in the incident radiation. The number of such spontaneous emissions per unit time per unit volume, is called **rate of spontaneous emission** which is propotional to only the number densityin the higher energy state, i.e, N_2.

$\therefore$ Rate of spontaneous emission = $A_{21} N_2$ ________ (2)

Where , A_{21} is the constant of proportionality called **Einstein coefficient of spontaneous emission.**

Case of Stimulated Emisssion: In the case of Stimulated emission, an external photon of appropriate frequency $\gamma = (E_2-E_1)/h$ is required to stimulate the atom for the corresponding downward transition, and thereby causing emission of stimulated photons, the energy density U_γ has a role to play in this case. The number of such stimulated emissions per unit time per unit volume, is called **rate of Stimulated emission** which is propotional to

a)the number density in the higher energy state, i.e, N_2 and

b)the energy density i.e, U_γ

$\therefore$ Rate of stimulated emission α $N_2 U_\gamma$

The Rate of stimulated emission $= B_{21} N_2 U_\gamma$ ———— (3)

Where , B_{21} is the constant of proportionality called **Einstein coefficient of stimulated emission.**

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$\therefore$ From Eqs (1), (2) & (3), we have, $B_{12} N_1 U_\gamma = A_{21} N_2 + B_{21} N_2 U_\gamma$

Or, $U_\gamma (B_{12}N_1 - B_{21}N_2) = A_{21}N_2$

Or, $$U_\gamma = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

By rearranging the above equation, we get,

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1}\right] \text{———— (4)}$$

But, by Boltzmann's law, we have, $\frac{N_2}{N_1} = e^{-\frac{(E_2-E_1)}{kT}} = e^{-\frac{h\gamma}{kt}}$

$$\therefore \quad \frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$$

$\therefore$ Eq (4) becomes, $U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\gamma}{kT}} - 1}\right]$ ——————(5)

According to Planck's law, the equation for U_γ is,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\frac{h\gamma}{kT}} - 1}\right] \text{——————(6)}$$

Now, comparing the equations (5) and (6) , term by term on the basis of positional identity, we have,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3},$$

And, $\frac{B_{12}}{B_{21}} = 1,$

Or, $B_{12} = B_{21},$

Which implies that the probability of induced absorption is equal to the probability of stimulated emission. Because of the above identity, the subscripts could be dropped, and A_{21} and B_{21} can be represented simply as A and B, and Eq (5) can be rewritten.

$\therefore$ At thermal equilibrium the equation for energy density is,

$$\boxed{U_\gamma = \frac{A}{B\left[e^{\frac{h\gamma}{kT}} - 1\right]}}$$

1.b) The ratio of the population of two energy states in a laser is 8.82x10^{-31}. If the temperature of the system is 57^0, what is the wavelength of the laser?

$$\frac{N_2}{N_1} = e^{-\left(\frac{hc}{\lambda kT}\right)}$$

$$\lambda = \frac{-hc}{KT}\left(\frac{1}{ln\left(N_2/_{N_1}\right)}\right)$$

$$= 631nm$$

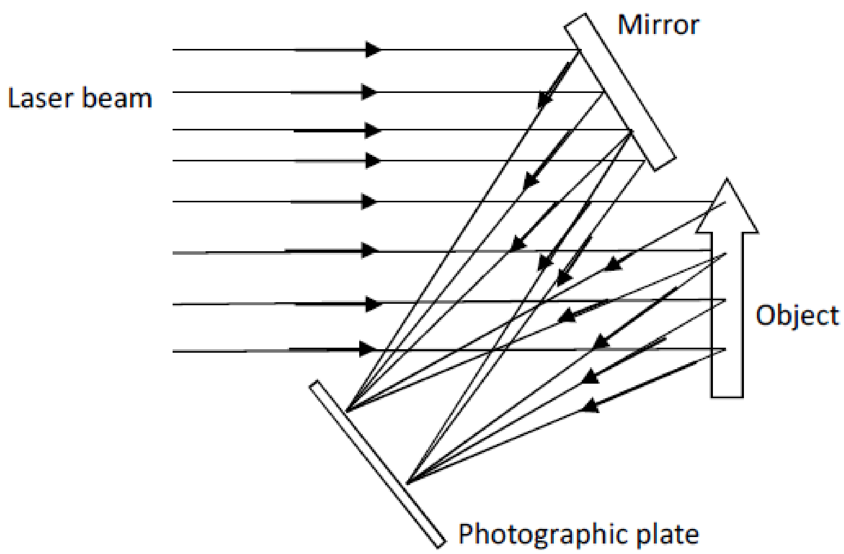
2.a) With neat ray diagrams, explain the construction and reconstruction of Hologram.

Holography:

The technique of producing three dimensional image of an object is known as holography.

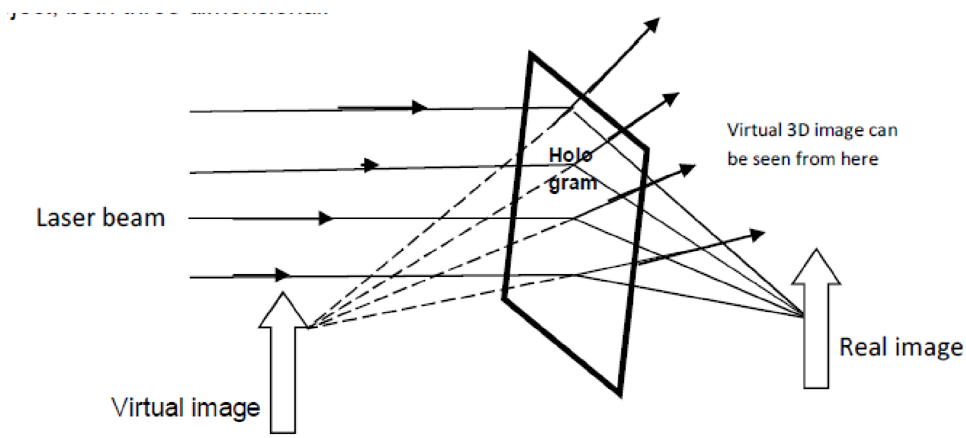
Recording of Hologram:

Light from a laser is partly reflected from the object and partly from a mirror as shown in the figure. They are made to interfere on the photographic plate so that interference pattern is formed on the plate. The photographic plate is developed and it is called the hologram. The hologram contains alternate dark and bright interference fringes.



Reconstruction of Image:

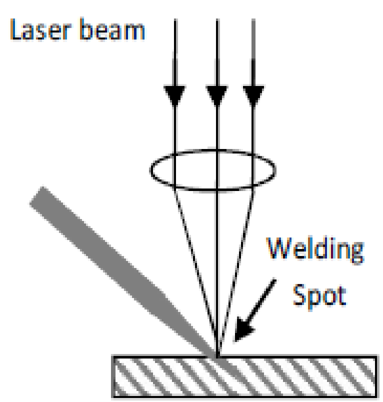
For reconstruction of the three dimensional image of the object, the hologram acts as a diffraction grating and when illuminated by a laser beam of same wavelength as the reference beam used for recording will result in the formation of a real image and a virtual image of the object which are three dimensional.



2. b) Describe the application of laser in welding, giving its advantages over conventional welding.

Welding:

In laser welding a high power laser beam is focused on to the spot to be welded. Due to the intense heat generated, the metal melts over the tiny area where the beam is focused and solidifies on cooling. Since the beam is focused to a sharp point the heat produced does not distort the welded region. It is a contactless process and hence there is no chance of any impurity getting into the welded joint. Moreover one can even weld difficult to reach locations in a material.



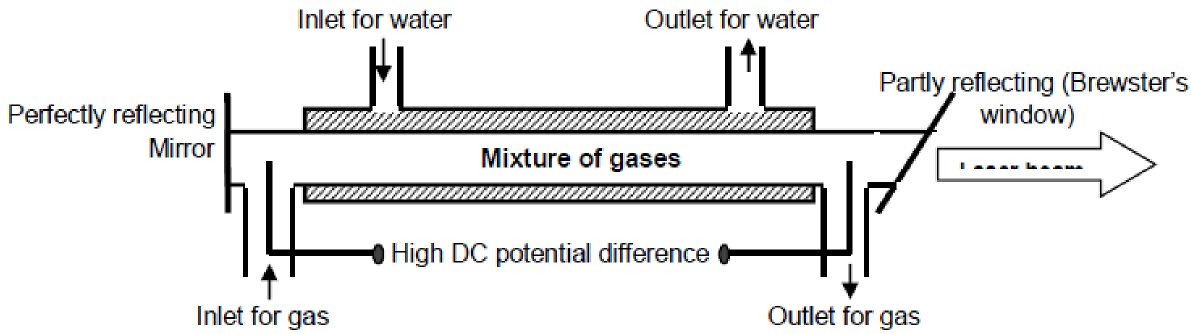
2. c) A semiconductor laser emits a radiation of wavelength 1.24 µm. What is its band gap value in eV?

$$E_g = \frac{hc}{\lambda}$$

$$E_g = 1.6X10^{-19}J = 1\ eV$$

3. a) Describe the construction of carbon dioxide laser and explain its working with the help of an energy level diagram. Give any two applications of a CO_2 laser.

Carbon dioxide LASER:



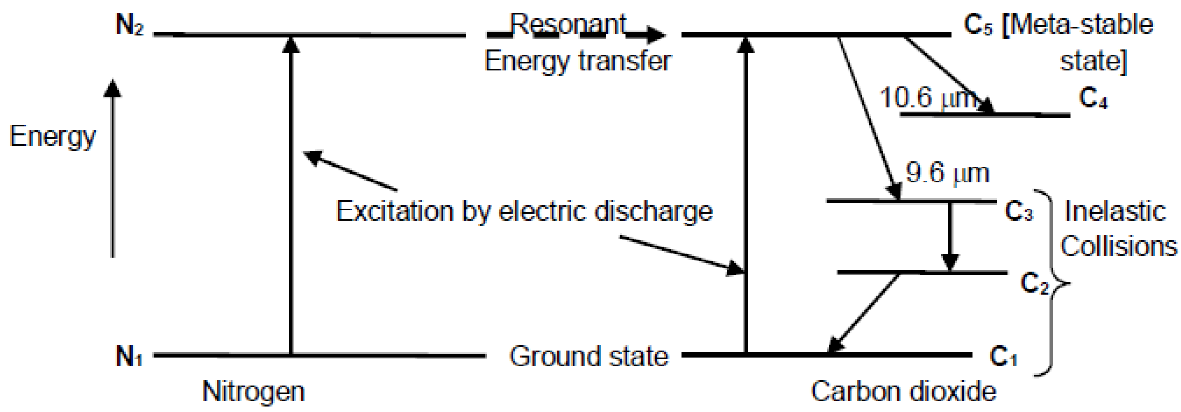
Schematic diagram of CO_2 laser

The carbon dioxide laser consists of a tube of about 5 m length and 2.5 cm diameter. The ends of the tube are closed with Brewster's windows. Beyond the windows are arranged mirrors of which one is perfectly reflecting and the other is partly reflecting. This forms the resonant cavity. The active medium in this laser is a mixture of CO_2, N_2 and He gases in the ratio 1:2:3. The pressure inside the tube is about 6-17 torr.

Pumping mechanism used here is electric discharge. When a high D. C. voltage is applied to the gas, electric discharge takes place through the gas mixture and N_2 and CO_2 atoms collide with the electrons and get excited to higher energy levels. For CO_2 gas, it so happens that there is a close coincidence in the energy of its C_5 state with the N_2 state of nitrogen gas. Therefore, when a N_2 molecule in the metastable state collides with a CO_2 molecule in the ground state, because of the matching of the energy levels, resonant transfer of energy takes place form N_2 molecule to a CO_2 molecule. As a result the CO_2 molecules get elevated to C_5 state whereas, the N_2 molecules return to the ground state. Thus the population of C_5 level of CO_2 increases rapidly which leads to population inversion with respect to the two lower lasing levels C_3 and C_4. The transition from level C_5 to C_4 produces laser of wavelength 10.6 µm and that from C_5 to C_3 results in a laser beam of wavelength 9.6 µm. Both these radiations lie in the IR region. The transitions from C_3 to C_2 and C_2 to C_1 happen by inelastic collision with helium atoms. The discharge tube containing the gas mixture is continuously cooled by circulating water as shown in the diagram.

The laser output is 100 kW in continuous wave mode and 10 kW in pulsed mode.

CO_2 laser finds application in welding, cutting and drilling. They are also used in communication systems.



Energy level diagram

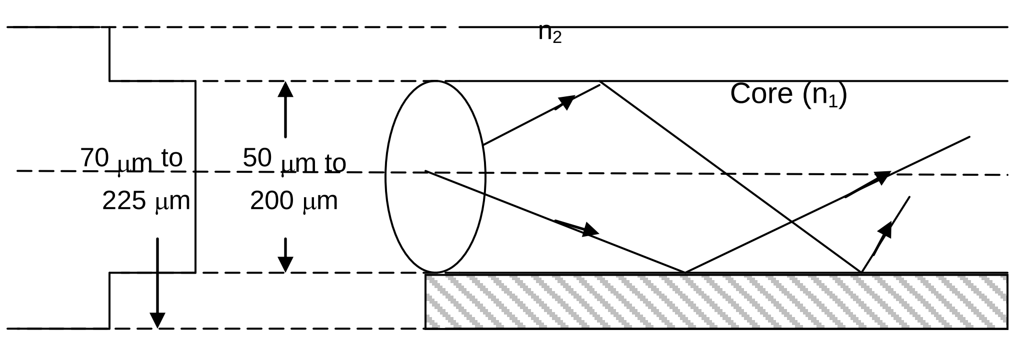
4. a) With neat diagrams, explain the salient features of the three different types of optical fibers with one application for each type.

Types of Optical Fires:

Depending upon the refractive index profile of the core and cladding, optical fibres are classified into two categories. They are: (1) the step index fibre and (2) the graded index fibre. Based on the number of modes accommodated, they are also classified as single mode fibres and multimode fibres.

Step Index Fibre:

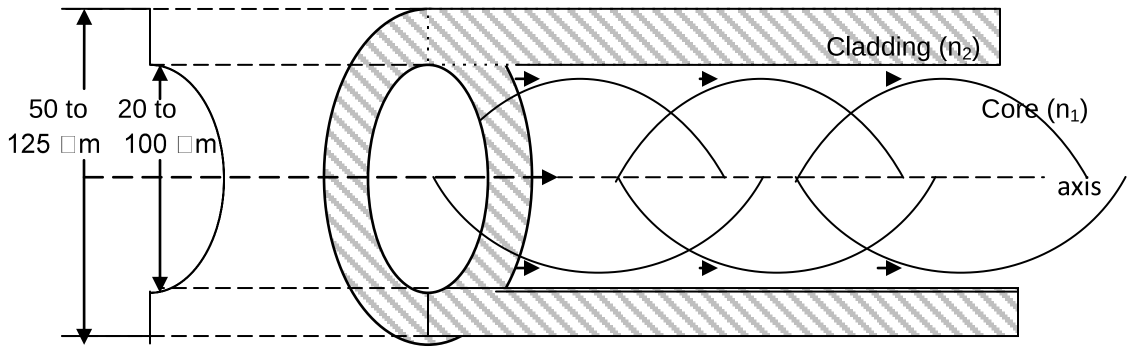
A step index fibre has a core of uniform refractive index n_1 and cladding of a lesser, but uniform, refractive index n_2. In other words there is an abrupt change in refractive index at the core-cladding interface. A fibre whose core diameter is in the range 50 µm to 200 µm can transmit a large number of modes. Thickness of cladding of such fibres is typically in the range 20 µm to 25µm. Such fibres are called multimode step index fibres. These fibres are not suitable for long distance communication because intermodal dispersion would cause signal distortion.



Step index fibres with core diameter in the range 2 to 10 µm are single mode fibres. They transmit only in the axial mode. Fibres used for long distance communication has core diameter in the range 25 to 30 µm.

Graded Index Fibre:

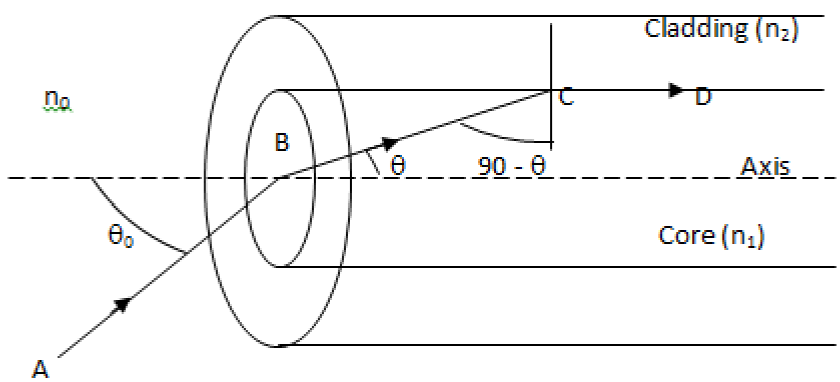
Graded index fibre has maximum refractive index along the axis of the core and gradually decreases radially up to the cladding and then remains constant throughout the cladding. Due to the gradually decreasing refractive index in the core, a ray that enters the core travels along a curved path. The core diameter of such fibres is usually in the range 20 to 100µm and thickness of the cladding is around 25 µm. These fibres are mostly used for medium range communication.



5. a) Obtain an expression for the numerical aperture of an optical fiber.

Expression for Numerical Aperture:

Consider a ray of light AB entering an optical fibre at an angle of incidence equal to the acceptance angle θ_0. It proceeds along BCD, as shown in figure.



Applying Snell's law for refraction at A,

$$n_0 \sin\theta_0 = n_1 \sin\theta$$

$$\text{Or, } \sin\theta_0 = \frac{n_1}{n_0}\sin\theta \quad \text{———————————————————} (1)$$

At B, the angle of refraction is 90°. Hence for refraction at B,

$$n_1 \text{ Sin } (90 - \theta) = n_2 \text{ Sin } 90$$

$$\text{Or, Cos } \theta = \frac{n_2}{n_1}$$

$$\therefore Sin\theta = \sqrt{1 - Cos^2\theta} = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$\text{Or, Sin } \theta = \frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_1}$$

Substituting this in eq. (1), we get,

$$\text{Sin } \theta_0 = \frac{n_1}{n_0} \times \frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_1} = \frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_0} \text{ -------------- } (2)$$

But, Sin θ_0 = numerical aperture (NA)

$$\therefore \text{ NA} = \frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_0}$$

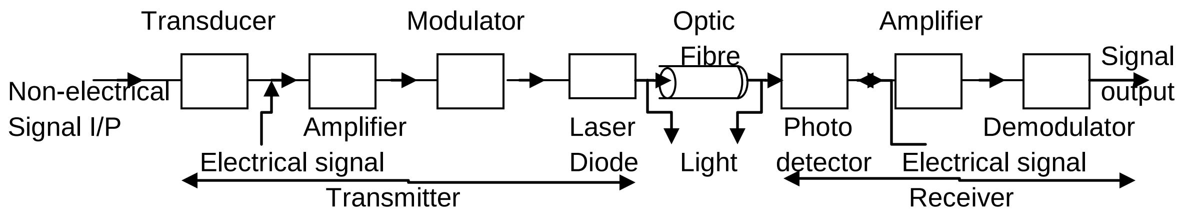
If the surrounding medium is air, $n_0 = 1$ and therefore,

$$\text{NA} = \sqrt{n_1^{\,2} - n_2^{\,2}}$$

5.b) Explain point to point communication system using optical fibers with the help of a block diagram.

Point to point Communication system using Optical Fibre:

Block diagram of an optical fiber communication system is shown below. The non-electrical signal such as sound, to be communicated over a long distance, is first converted into an electrical signal using a transducer (say, microphone). The electrical signal is then amplified using electronic amplifiers and modulated with a high frequency electrical signal. The modulated signal is then fed to a laser diode, which converts the signal into a corresponding optical signal. The light signal is transmitted through an optic fibre. At the receiver, the light signal is fed to a photo-detector, which converts it into anelectrical signal. This is amplified, demodulated and then fed to a transducer (say, loud speaker) which converts the electrical signal back to the original sound signal. If it is transfer of digital data from one computer to another, then there is no need for the transducer, as the input and output signals are both electrical.



(Bock diagram of Optical Fibre Communication System)

5.C) A signal with input power 200 mW loses 10% of its power after travelling a distance of 3000 m. Find the attenuation coefficient of the fiber.

Given: $P_i = 200\text{mW} = 200\text{x}10^{-3}\text{w}$, $P_o = 90\% \, P_i = 0.9\text{x}200\text{x}10^{-3}\text{w}$;

$L = 3000 \text{ m} = 3 \text{ km}$; $\alpha = ?$,

$$\text{Using } \alpha = -\frac{10}{L}\log\left[\frac{P_o}{P_i}\right] = -\frac{10}{3}\log\left[\frac{0.9\text{x}200\text{x}10^{-3}}{200\text{x}10^{-3}}\right] = 0.1525 \text{ dB/km}$$

6. a) What is attenuation? Explain any two factors contributing to the fiber loss?

Attenuation: Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber.

Causes of Attenuation: Attenuation in optical fibres is caused by:

1) **Scattering**: Impurities and structural inhomogeneity in the fibre could cause scattering of light passing through it, resulting in loss of energy of the signal. The scattering involved is Rayleigh's scattering, which depends on λ^{-4} and therefore is more for smaller wavelengths.

2) **Radiation Loss**: While laying fibre optic cables they have to be bent at corners, which result in changes in angle of incidence at the core-cladding interface. This results in partial loss of light by refraction, as shown in figure. Loss also happens when the core-cladding interface has microscopic irregularities. This kind of loss is called radiation loss.

6.b) Explain the terms (i) Acceptance angle (ii) Fractional index change (iii) V-number

Acceptance angle:

Acceptance angle is the maximum angle submitted by the ray with the axis of the fibre so that light can be accepted and guided along the fibre.

Fractional Index Change:

The ratio of the difference between refractive indices of core and cladding to that of core is called fractional refractive index change.

$$\Delta = \frac{(n_1 - n_2)}{n_1}$$

V-number: The number of modes supported for propagation in the fiber is determined by a parameter called V-number.

$$V = \frac{\pi d}{\lambda} X \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$$

$$V = \frac{\pi d}{\lambda} X \ NA$$

5. b) An optic glass fiber of refractive index 1.5 is to be clad with another glass to ensure total internal reflection that will contain light travelling within 5^0 of the fiber axis. What maximum index of refraction is allowed for the cladding?

For grazing incidence,

$$n_1 \sin 85^0 = n_2 \sin 90^0$$

$$n_2 = 1.49$$

the maximum index of refraction allowed for cladding should be < 1.49

7. a) What are the similarities and differences between the classical free electron theory and quantum free electron theory?

Similarities between Classical and Quantum Free Electron Theories:

1. According to both the theories conduction is due to free electrons

2. The attraction between electrons and positive ion cores and the repulsion between electrons themselves is neglected.

3. The electrons are assumed to be confined to the metal by potential barriers at the boundaries and the potential inside the metal is assumed to be constant.

Differences between Classical and Quantum Free Electron Theories:

	Classical theory	Quantum theory
1	Energy distribution of electrons is according to Maxwell – Boltzmann statistics	Energy distribution of electrons is according to Fermi – Dirac statistics
2	Electrons can have continuous energy values	Electrons have discrete and quantized energies
3	Many electrons can have the same energy	No two electrons can have the same energy

7. b) Define the terms, (i) Mean Collision time (ii) Relaxation time

Mean collision time (τ):

The average time interval between successive collisions of the free electrons with the lattice ions is called mean collision time. If v is the average velocity of electrons and λ is the mean free path, then $\tau = \lambda/v$.

Relaxation time (τ_r) :

When the electric field applied across a conductor is switched off, the drift velocity decreases exponentially to zero. The time during which the drift velocity reduces to 1/e times the initial value is called relaxation time.

7. c) Find the relaxation time for conduction electrons in aluminum metal.Given that for aluminum, the electron concentration=18 x10^{22}/cm^3 and resistivity = 2.7x10^{-8}Ωm.

$$\text{Using } \rho = \frac{m}{ne^2\tau} \text{ ,we get } \tau = \frac{m}{\rho ne^2}$$

$$\tau = 7.3X10^{-15}s$$

8. a) Using the free electron model, derive an expression for electrical conductivity in metals.

The drift velocity of electron $v_d = \frac{e\tau E}{m}$ ......(1)

Also from Ohm's law electrical conductivity , $\sigma = \frac{j}{E}$ ..(2)where j=current density & E=electric field But $j = nev_d$ ... (3)

$\therefore$ eqns 1&3 ,we get $j = ne.\frac{e\tau E}{m}$

$$\therefore \frac{j}{E} = \frac{ne^2\tau}{m} \text{....(4)}$$

From eqns 2 &4,we get $\sigma = \frac{ne^2\tau}{m}$ .....(5)

8. b) Discuss any two merits of quantum free electron theory.

Merits of Quantum Free Electron Theory:

1. Specific heat of free electrons:

From quantum theory of free electrons, the specific heat of free electrons is given by

Cv = (2k/EF) RT

For a typical value of EF = 5 eV, we get Cv = 10^{-4}RT.

Hence gives the correct molar specific heat at constant volume for free electrons in metals.

2. Temperature dependence of conductivity of metals:

The quantum free electron theory gives the correct temperature dependence of conductivity.

From the equation for conductivity according to Q.F.E.T, $\sigma \propto \lambda$

But, $\lambda \propto \frac{1}{\pi r^2}$

Where 'r' is the amplitude of lattice vibrations which is directly proportional to temperature i.e.

$$r^2 \propto T$$

$$\therefore \lambda \propto \frac{1}{T}$$

Since, $\sigma \propto \lambda$

$$\therefore \sigma \propto \frac{1}{T}$$

8.C) The Fermi level for a metal is 3.1 eV, calculate the energies for which the probability of occupancy at 300 k is 98% and 50%.

$$\text{Using } f(E) = \frac{1}{1+e^{(E-E_F)/kT}},$$

$$\text{we get } E = E_F + kT\, In\, [\frac{1}{f(E)} - 1]$$

E_1=4.7X10^{-19}J=2.99eV

E_2= 4.94X10^{-19}J=3.09eV

9. a) Define Fermi energy and Fermi velocity. Discuss the variation of Fermi factor with temperature and energy.

Fermi Energy E_F:

The energy corresponding to the highest occupied level in the valence band at 0 K is called Fermi energy.

Fermi Velocity (V_F): The velocity of the electrons which occupy the Fermi level is called the Fermi Velocity (V_F).

$$V_F = \sqrt{\frac{2E_F}{m}}$$

Fermi factor: Under thermal equilibrium the free electrons occupy various energy levels in accordance with a statistical rule known as Fermi – Dirac statistics. Fermi – Dirac statistics enables the evaluation of probability of finding electron in energy levels over a certain range of energy values. The evaluation is done with the help of a quantity called **Fermi factor** f (E) given by:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

Effect of temperature on Fermi function

Case 1: Probability of occupation for E< E_F at T= 0k.

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1+e^{-\infty}}$$

$$f(E) = \frac{1}{1 + 0}$$

$$f(E) = 1$$

The distribution $f(E) = 1$ means that at T = 0k, all the energy levels below the Fermi level are fully occupied by electrons leaving the upper levels vacant.

I.e. there is 100 % probability of finding an electron below the Fermi energy level.

Case 2: Probability of occupation for E >E_F at T= 0k.

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1 + e^{\infty}}$$

$$f(E) = \frac{1}{1 + \infty}$$

$$f(E) = \frac{1}{\infty} = 0$$

This indicates that at T= 0k, the energy levels above the Fermi level is not occupied by electrons, they are vacant.

I.e. there is 0% probability of finding the electron above the Fermi level at absolute zero K

Case 3: Probability of occupation at ordinary temperatures.

At temperatures above 0 K and E = E_F .

Substituting this condition in F-D equation, we get

$$f(E) = \frac{1}{1 + e^{0}}$$

$$f(E) = \frac{1}{1 + 1}$$

$$f(E) = \frac{1}{2} = 0.5$$

At temperatures above 0K, there is only 50 % probability for an electron to occupy Fermi level.

9. b) For intrinsic gallium arsenide, the room temperature conductivity is $10^{-6}/\Omega m$. The electron and hole mobility's are respectively 0.85 m^2/Vs and 0.04 m^2/Vs. Calculate the intrinsic carrier concentration.

Given: $\sigma_i = 10^{-6}\ /\Omega\text{m}\ ;\ \mu_e\ = 0.85\ m^2/Vs\ \mu_e\ = 0.04\ m^2/Vs;\ n_i\ =?$

$Using, \sigma_i = n_i e\ (\mu_e\ + \mu_h),$

$$we\ get\ , n_i = \frac{\sigma_i}{e\ (\mu_e\ + \mu_h)}$$

$$n_i = \frac{10^{-6}}{1.6x10^{-19}\ (0.85+0.04)}$$

$$= 7.023x10^{12}/m^3$$

10. a) State law of mass action for a semiconductor. Derive an expression for electrical conductivity of an intrinsic semiconductor.

Law of mass action states that the product of the concentration of charge carriers electrons (n_e)and holes(n_h) in an intrinsic semiconductor is equal to the square of the intrinsic charge carriers(n_i^2) at any temperature.

ie: $\boldsymbol{n_i^2 = n_e . n_h}$

The current in a semiconductor is contributed by both electrons and holes, moving in opposite directions. Consider a semiconductor with area of cross-section, A, free electron concentration n_e and number density of holes, n_h. The current due to free electrons is:

$$\mathbf{I_e = n_e\ A\ e\ v_e,}$$

Where e is electronic charge and v_e is the drift velocity of electrons.

And, the current due to holes is:

$$\mathbf{I_h = n_h\ A\ e\ v_h,}$$

Where v_h is the drift velocity of holes.

Hence the total current, $\mathbf{I = I_e + I_h = n_e\ A\ e\ v_e + n_h\ A\ e\ v_h = A\ e\ (n_e\ v_e + n_h\ v_h)}$

$\therefore$ Current density $\mathbf{J = I/A = e\ (n_e\ v_e + n_h\ v_h)}$

But, current density, $J = \sigma E$, where σ is conductivity and E, the field across the semiconductor.

$$\therefore \quad \sigma E = e\ (n_e\ v_e + n_h\ v_h)$$

$$\therefore \quad \sigma = e\ (n_e\ v_e/E + n_h\ v_h/E)$$

$$\text{Or,} \quad \boldsymbol{\sigma = e\ (n_e\ \mu_e + n_h\ \mu_h)}$$

Where, $v_e/E = \mu_e$ is the mobility of electrons and $v_h/E = \mu_h$ is the mobility of holes.

For intrinsic semiconductors $n_e = n_h = n_i$, where n_i is the density of intrinsic charge carriers. Hence for *intrinsic semiconductors*, $\boldsymbol{\sigma = e\, n_i (\mu_e + \mu_h)}$

10. b) Find the temperature at which there is 1 % probability that a state with energy 0.5 eV above Fermi level will be occupied.

Given: $(E - E_F) = 0.5\ eV = 0.5x1.6x10^{-19}$ J , k=$1.38x10^{-23}$ J/K ; f(E) = 1 % = 1/100 =0.01 ; T= ?

Using $f(E)=\frac{1}{1+e^{(E-E_F)/kT}}$,we get $T=\frac{(E-E_F)}{k.In(\frac{1}{f(E)}-1)}$

$$=\frac{0.5x1.6x10^{-19}}{1.38x10^{-23}.In(\frac{1}{0.01}-1)}$$

$$=\frac{0.5x1.6x10^{-19}}{1.38x10^{-23}.In(100-1)}$$

$$= 1262\ K = 1.262x10^{3}\ K$$