P.E.S. Institute of Technology – Bangalore South Campus
Department of Science & Humanities
2nd Internal Assessment Test (Scheme & Solutions)
Even Semester – JAN 2018
SUBJECT / CODE: 17CIV23

CONTINUOUS INTERNAL EVALUATION TEST -2

Date : **04/04/2018**		Marks:**60**
Subject & Code : **Elements of Civil Engg & Mechanics/ 17CIV23**		Section: **A,B,C,D & E**
Name of faculty : Prof. D Jawahar, Prof. Rashmi B A & Prof. Sachna K G		Time : **8:30 to 10:00a.m**

Note: Answer FIVE full questions choosing any ONE full question from each part.

		PART 1	
1	**a**	What is the moment of a force? What are the various moments encountered in practice? Explain them	**4**
	b	The forces acting on a dam are as shown in the fig 1.b. Determine the resultant force acting on the dam & also calculate the point of intersection of the resultant with the base.	**8**
2	**a**	State and prove Varignon's principle of moments.	**6**
	b	Define a couple and explain its characteristics	**6**
		PART 2	
3	**a**	A flat plate is subjected to the coplanar system of forces as shown in fig.3.a. Each square of the inscribed grid is having a length of 1m. Determine the resultant, its x-intercept, y-intercept & distance w r t 'O'.	**12**
4	**a**	A 100 N vertical force is applied to the end of a lever which is attached to a shaft as shown in fig 4.a. Determine i) The moment of force about O. ii) The horizontal force applied at 'A' which creates same moment about 'O'. iii) The smallest force applied at 'A' which creates same moment about 'O'	**6**
	b	Determine the moment of inertia of the area shown in the fig 4.b about the axis AB (all the dimensions are in mm)	**6**
		PART 3	
5	**a**	Determine the horizontal force P applies to the lower block to just pull it to the right as shown in the fig 5.a. The coefficient of friction between the blocks is 0.2 and that between the lower block and the plane is 0.25. Also find the tension in the string	**12**
6	**a**	Two blocks A & B weighing 20N each rest on a rough inclined plane and are connected by a string as shown in the fig 6.a. The coefficient of friction between the block A & the plane is 0.2, while that for block B & the plane is 0.3, find i) The angle of inclination of the plane for which sliding will impend ii) Tension in the string	**12**
		PART 4	
7	**a**	Explain the terms i) Angle of friction ii) Angle of repose	**4**
	b	Explain i) Types of friction ii) laws of dry friction	**8**
8	**a**	Derive an expression for moment of inertia of a triangle about the base using the method of integration. Hence find moment of inertia about the centroidal axis parallel to base.	**12**
		PART 5	
9	**a**	Determine the moment of inertia of the area shown in the fig 9.a about the axis AB and PQ	**12**
10	**a**	Determine the second moment of the area about the horizontal centroidal axis shown in the fig 10.a. Also find the radius of gyration	**12**

Fig 1.b

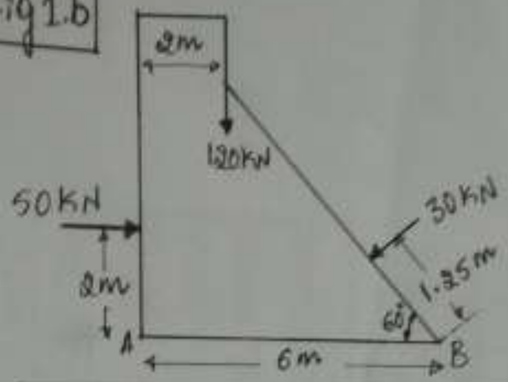


Fig 3a

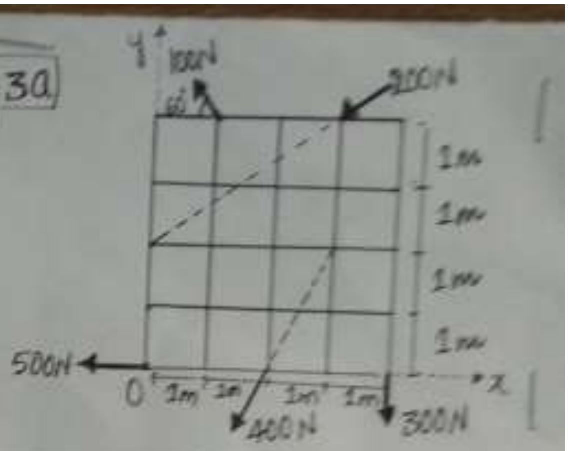


Fig.4.a

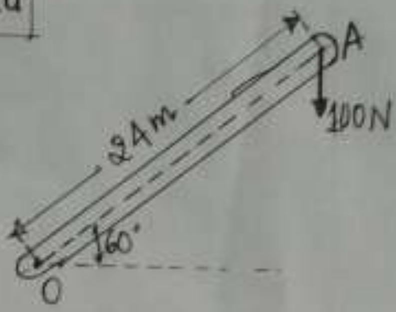


Fig.4.b

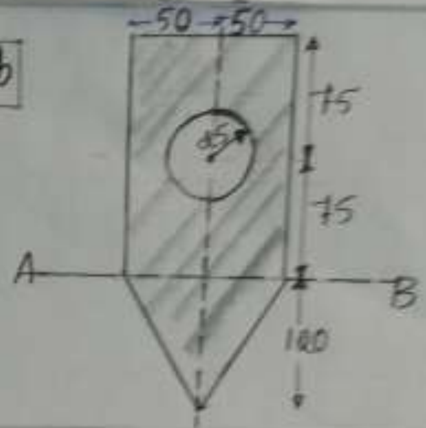


Fig 5.a

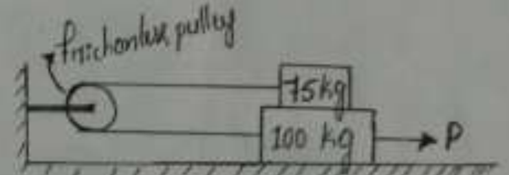


Fig.6.a

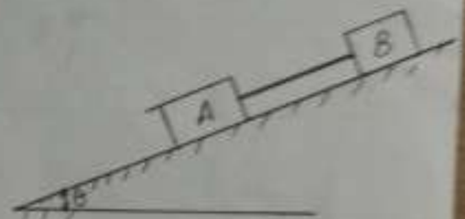
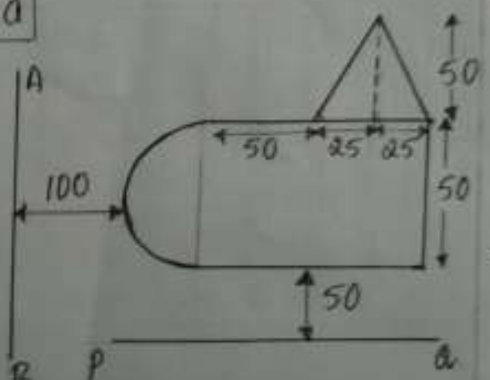
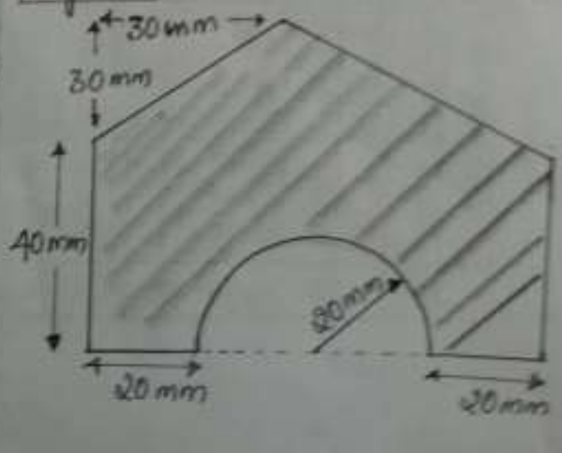


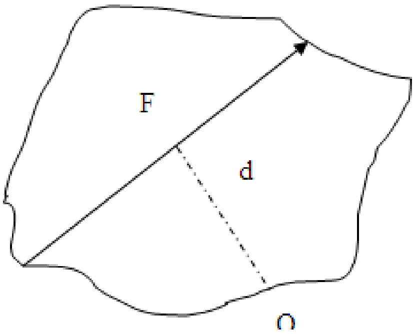
Fig 9.a



* All dimensions are in mm

Fig 10.a



Q. NO			
1	a	Moment of a force is defined as the product of the magnitude of the force and perpendicular distance of the point from the line of action of the force from the point. Let “F” be a force acting in a plane. Let” O” be a point or particle in the same plane. Let “d” be the perpendicular distance of the line of action of the force from the point “O”. Thus the moment of the force about the point “O” is given as Mo= F x d Moment or rotational effect of a force is a physical quantity dependent on the units for force and distance. Hence the units for moment can be “Nm” or “KNm” or “ N mm” etc.  	4
	b		8

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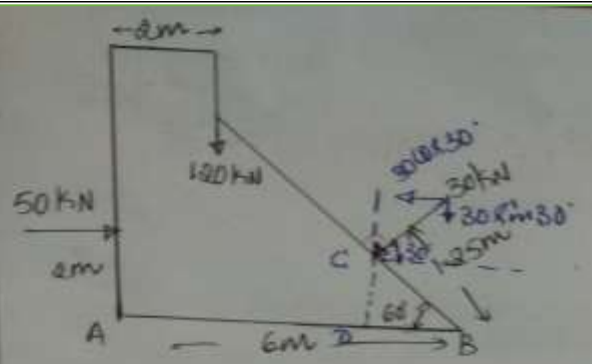


Δ^{le}

CB = 1.25 m

CD = 1.25 sin 60° = 1.082 m

DB = 1.25 cos 60° = 0.6 m

AD = AB − DB
= 5.4 m

$\Sigma F_x = +50 - 30\cos 30$

$\boxed{\Sigma F_x = 24.02\ kN}$

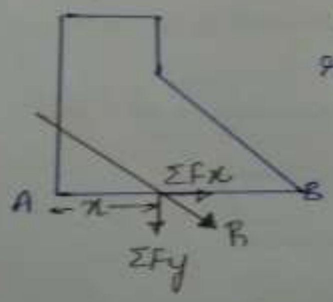
$\Sigma F_y = -120 - 30\sin 30$

$\boxed{\Sigma F_y = -135\ kN}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} \Rightarrow \boxed{R = 137.12\ kN}$

$\theta = \tan^{-1}(\Sigma F_y / \Sigma F_x) \Rightarrow \boxed{\theta = 79.9}$

Since ΣF_x is +ve & ΣF_y is −ve resultant lie in 4th quadrant



Let x be the horizontal intercept of resultant from A.

$\Sigma M_A = +(120\times 2) + (50\times 2) + 30\sin 30\,(AD) - 30\cos 30\,(CD)$

$\Sigma M_A = 392.9\ kN\text{-}m$

$\therefore x = \frac{|\Sigma M_A|}{|\Sigma F_y|} = \frac{392.9}{135} = 2.90\ m$
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Statement: If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.
PROOF: For example, consider only two forces F1 and F2 represented in magnitude and direction by AB and AC as shown in figure below.
Let O be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.
By the parallelogram law of forces, the diagonal AD represents, in magnitude and Direction, the resultant of two forces F1 and F2, let R be the resultant force. By geometrical representation of moments
the moment of force about O=2 Area of triangle AOB
the moment of force about O=2 Area of triangle AOC
the moment of force about O=2 Area of triangle AOD
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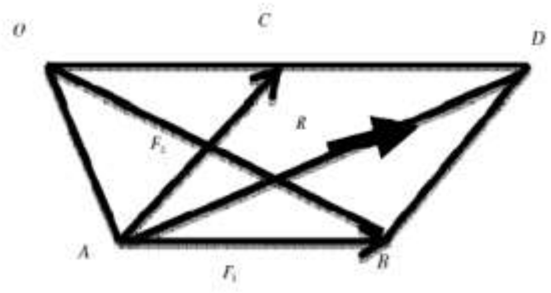
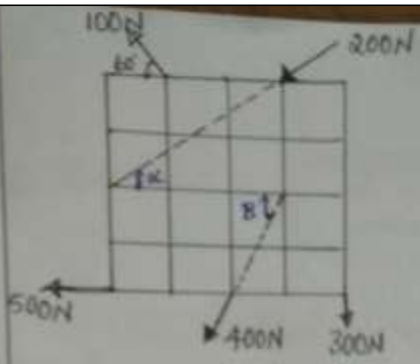
		But, Area of triangle AOD=Area of triangle AOC + Area of triangle ACD Also, Area of triangle ACD=Area of triangle ADB=Area of triangle AOB Area of triangle AOD=Area of triangle AOC + Area of triangle AOB Multiplying throughout by 2, we obtain 2 Area of triangle AOD =2 Area of triangle AOC+2 Area of triangle AOB i.e., Moment of force R about O=Moment of force F1 about O + Moment of force F2 about O Similarly, this principle can be extended for any number of forces. 	
	b	**Couple:** Two forces of same magnitude separated by a definite distance, (acting parallely) in a opposite direction are said to form a couple. **Characteristics of Couple:** 1. The algebraic sum of the forces constituting the couple is zero. 2. The algebraic sum of the moments of the forces, constituting the couple about any point is same. 3. A couple cannot be balanced by a single force, but can be balanced only by a couple; but of opposite sense.	**6**

3 **a**



$\alpha = \tan^{-1}(2/3) = 33.69$

$\beta = \tan^{-1}(2/1) = 63.43$

$\Sigma F_x = -500 - 100\cos 60^\circ - 400\cos 63.43 - 200\cos 33.69$

$\boxed{\Sigma F_x = -895.33\,N}$

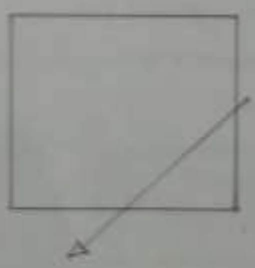
$\Sigma F_y = +100\sin 60 - 200\sin(33.69) - 400\sin(63.43) - 300$

$\boxed{\Sigma F_y = -682.09\,N}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} \Rightarrow \boxed{1125.55\,N = R}$

$\theta = \tan^{-1}\left(\frac{|\Sigma F_y|}{|\Sigma F_x|}\right) = 37.30$

Since ΣF_x is -ve & ΣF_y is -ve the resultant lies in 3rd quadrant.



$M_o = -(100)\cos(60)(4) - 100\sin(60)(1) - 200\cos(33.69)(2) + 400\cos\sin 63.43(2) + 300(4)$

$M_o = -200 - 86.6 - 332.82 + 715.5 + 1200$

$\boxed{M_o = 1296.08\ N\text{-}m}$

$x\text{-intercept} = \frac{|\Sigma M_o|}{|\Sigma F_y|} = 1.90\,m$

$y\text{-intercept} = \frac{M_o}{\Sigma F_x} = 1.447\,m$

$d = \frac{M_o}{R} = 1.15\,m$

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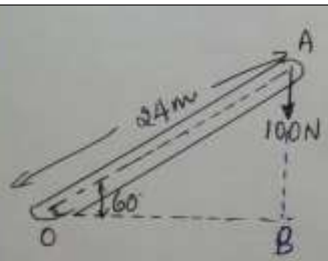
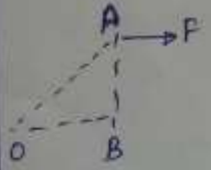
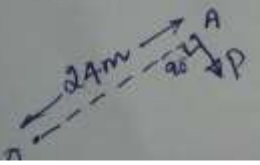


From the Δ^{le} AOB ③
$OA = 24m$
$BO = 24 \cos 60 = 12m$
$AB = 24 \sin 60 = 20.785m$
(i) The moment of force about 'O'
$M_O = (100)(OB) = 1200$ N-m
(ii) Let F be the horizontal force applied at 'A' giving same moment $M_O = 1200$ N-m


$M_O = F \times AB$
$1200 = F \times 20.785$
$\boxed{F = 5.77N}$
(iii) Let 'P' be the smallest force applied at A which acts perpendicular to line OA


$M_O = P \times OA$
$1200 = P \times 24$
$\boxed{P = 50N}$
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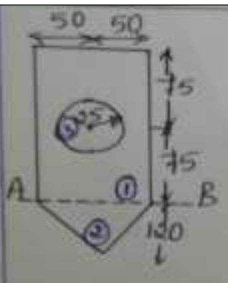

Moment of Inertia of shaded area about AB $\Big\} = \Big\{$ MI of fig ① about AB + MI of fig ② about AB − MI of fig ③
MI of rectangle about its base AB $\Big\} = I_{AB}① = \frac{bd^3}{3} = 1.125 \times 10^8 mm^4$
MI of Δ^{le} about its base AB $\Big\} = I_{AB}② = \frac{bh^3}{12} = 14.40 \times 10^6 mm^4$
MI of a $\bigcirc^{le}$ about AB $\Big\} = \frac{\pi R^4}{4} + (\pi R^2)(75)^2 = 11.35 \times 10^6 mm^4$
$I_{AB} = I_{AB}① + I_{AB}② - I_{AB}③$
$\boxed{I_{AB} = 115.55 \times 10^6 mm^4}$
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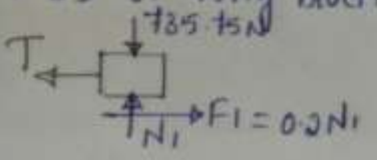
5	a	(handwritten solution below)	12

F.B.D of 75kg block



$W = 75 \times 9.81 = 735.75\,N$

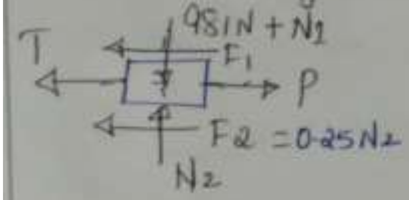
$\Sigma F_y = 0\ (\uparrow +ve)$

$-735.75 + N_1 = 0 \Rightarrow \boxed{N_1 = 735.75\,N}$

$F_1 = 147.15\,N$

$\Sigma F_x = 0 \Rightarrow -T + F_1 = 0 \quad \boxed{T = 147.15\,N}$

F.B.D of 100kg block



$W = 100 \times 9.81 = 981\,N$

$\Sigma F_y = 0\ (\uparrow +ve)$

$+N_2 - 981 - 735.75 = 0$

$\boxed{N_2 = 1716.75\,N}$

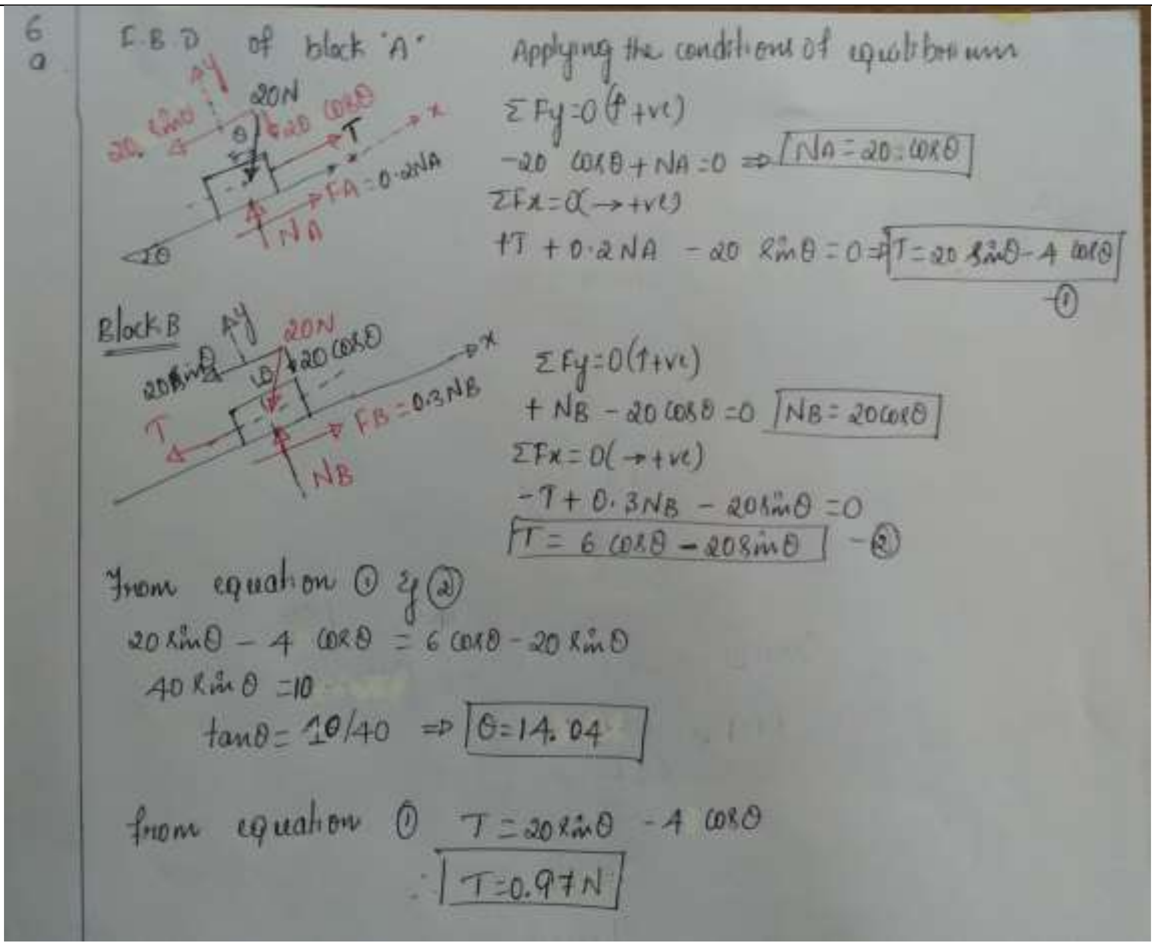
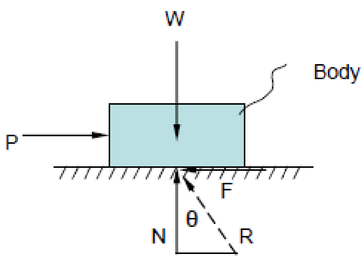
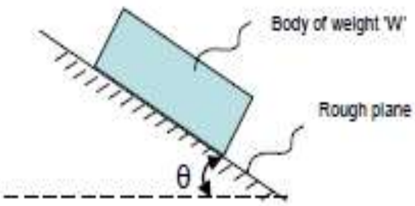
$\Sigma F_x = 0\ (\rightarrow +ve)$

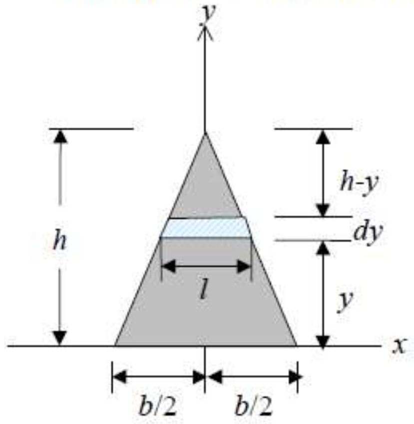
$-T + P - 0.25N_2 - F_1 = 0$

$P = T + 0.25N_2 + F_1$

$= 147.15 + 0.25(1716.75) + 147.15$

$\boxed{P = 723.48\,N}$

6	**a**	 F.B.D of block 'A' Applying the conditions of equilibrium $\Sigma F_y = 0\ (\uparrow +ve)$ $-20\cos\theta + N_A = 0 \Rightarrow \boxed{N_A = 20\cos\theta}$ $\Sigma F_x = 0\ (\rightarrow +ve)$ $+T + 0.2N_A - 20\sin\theta = 0 \Rightarrow \boxed{T = 20\sin\theta - 4\cos\theta}$ —(1) Block B $\Sigma F_y = 0\ (\uparrow +ve)$ $+N_B - 20\cos\theta = 0\quad \boxed{N_B = 20\cos\theta}$ $\Sigma F_x = 0\ (\rightarrow +ve)$ $-T + 0.3N_B - 20\sin\theta = 0$ $\boxed{T = 6\cos\theta - 20\sin\theta}$ —(2) From equation (1) & (2) $20\sin\theta - 4\cos\theta = 6\cos\theta - 20\sin\theta$ $40\sin\theta = 10$ $\tan\theta = 10/40 \Rightarrow \boxed{\theta = 14.04}$ from equation (1) $T = 20\sin\theta - 4\cos\theta$ $\therefore \boxed{T = 0.97N}$	**12**
7	**a**	**i) Angle of Friction**  Consider a body weighing "W" placed on a horizontal plane. Let "P" be an applied force required to just move the body such that, frictional resistance reaches limiting friction value. Let "R" be resultant of F & N. Let "θ" be the angle made by the resultant with the direction of N. such an angle "θ" is called the Angle of friction.  ii) **Angle of Repose:** Consider a body weighing „w" placed on a rough inclined plane, which makes an angle „θ" with the horizontal. When „θ" value is small, the body is in equilibrium or rest without sliding. If „θ" is gradually increased, a stage reaches when the body tends to slide down the plane The maximum inclination of the plane with the horizontal, on which a body free from external forces can rest without sliding is called angle of repose.	**4**
	b	**i) Types of Friction** The friction experienced by a body when it is at rest or in equilibrium is known as **static friction.** It can range between a zero to limiting fraction value. The friction experienced by a body when it is moving is called **dynamic friction**. The dynamic friction experienced by a body as it slides over a plane is called **sliding friction**.	**12**

		The dynamic friction experienced by a body as it roles over surface as shown in figure is called **rolling friction.** The frictional resistance developed between bodies having dry surfaces of contact obey certain laws called laws of **dry friction.** **ii) Laws Of Dry Friction: (Columb's Laws)** 1) The frictional resistance depends upon the roughness or smoothness of the surface. 2) Frictional resistance acts in a direction opposite to the motion of the body. 3) The frictional resistance is independent of the area of contact between the two bodies. 4) The ratio of the limiting friction value (F) to the normal reaction (N) is a constant (co-efficient of friction, μ) 5) The magnitude of the frictional resistance developed is exactly equal to the applied force till limiting friction value is reached or where the bodies is about to move.	
8	**a**	**• Moment of Inertia of a Triangular Area.**  $dI_x = y^2\,dA \qquad dA = l\,dy$ Using similar triangles, we have $\frac{l}{b} = \frac{h-y}{h} \qquad l = b\frac{h-y}{h} \qquad dA = b\frac{h-y}{h}dy$ Integrating dI_x from $y = 0$ to $y = h$, we obtain $I_x = \int y^2\,dA$ $= \int_0^h y^2 b\frac{h-y}{h}dy = \frac{b}{h}\int_0^h (hy^2 - y^3)dy$ $= \frac{b}{h}[h\frac{y^3}{3} - \frac{y^4}{4}]_0^h = \frac{bh^3}{12}$ $I_x = \bar{I}_x + Ad^2$ $\bar{I}_x = I_x - Ad^2$ $= \frac{bh^3}{12} - (\frac{bh}{2})(\frac{h}{3})^2 = \frac{bh^3}{36}$	**12**

9

12

9(a)

I_{PQ} = Sum of the moment of inertia of { ① semicircle ② rectangle ③ triangle } about PQ

① MI of semicircle about PQ $= \left(\frac{\pi R^4}{8}\right) + \left(\frac{\pi R^2}{2}\right)\left[25+50\right]^2$ Here $R = 25$

$= 56.76 \times 10^5\ mm^4$

② MI of rectangle about PQ $= \frac{bd^3}{12} + (bd)\left[25+50\right]^2$ Here $b = 100$, $d = 50$

$= 291.7 \times 10^5\ mm^4$

③ MI of triangle about PQ $= \frac{bh^3}{36} + \left(\frac{1}{2}b.h\right)\left[50+50+\frac{50}{3}\right]^2$

$= 171.8 \times 10^5\ mm^4$

$\therefore I_{PQ} = (56.76 + 291.7 + 171.8)\ 10^5$

$$\boxed{I_{PQ} = 52.026 \times 10^6\ mm^4}$$

I_{AB} = Sum of the MI of { ① semicircle ② rectangle ③ triangle } about AB

① MI of semicircle about AB $= 0.11R^4 + \frac{\pi R^2}{2}\left[100+25-\frac{4R}{3\pi}\right]^2$ Here $R = 25$ mm

$= 128.89 \times 10^5\ mm^4$

② MI of rectangle about AB $= \frac{db^3}{12} + bd\left[100+25+\frac{100}{2}\right]^2$

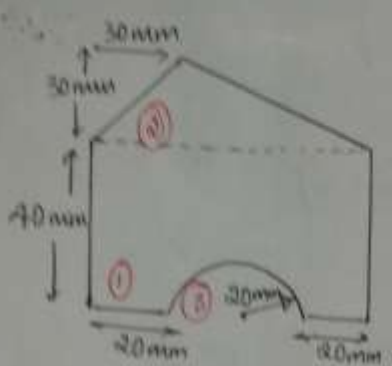
$= 1573.0 \times 10^5\ mm^4$

③ MI of triangle about AB $= \left(\frac{hb^3}{48}\right) + \frac{1}{2}bh\left[100+25+50+25\right]^2$ $h = 50$, $b = 50$

$= 501.3 \times 10^5\ mm^4$

$\therefore I_{AB} = (128.89 + 1573.0 + 501.3) \times 10^5$

$$\boxed{I_{AB} = 2.203 \times 10^8\ mm^4}$$

10	**a**	 ① $\square$ $d=40$, $b=80$ $a_1 = bd = 3200$ $y_1 = d/2 = 20$ ② $\triangle$ $h=30$, $b=80$ $a_2 = \frac{1}{2}(80)(30) = 1200$ $y_2 = 40 + \frac{30}{3}$, $y_2 = 50\,mm$ Deduction ③ semicircle $R=20$ $a_3 = \frac{-\pi R^2}{2} = -628.32$ $y_3 = \frac{4R}{3\pi}$, $y_3 = 8.488$ $\bar{Y} = \frac{\Sigma ay}{A} \Rightarrow \frac{118666.82}{3771.68} = 31.46\,mm$ $I_{xx} = \Sigma(I_{xx} + A \cdot y_c^2)$ $= \frac{bd^3}{12} + a_1(\bar{Y}-y_1)^2 + \left[\frac{bh^3}{36} + a_2[\bar{Y}-y_2]^2\right] - \left[0.11R^4 + a_3[\bar{Y}-y_3]^2\right)$ $= \left\{\frac{80(40)^3}{12} + 3200[31.46-20]^2\right\} + \left\{\left[\frac{80(30)^3}{36} + 1200[31.46-50]^2\right)\right\}$ $- \left\{0.11(20)^4 + 628.32(31.46-8.488)^2\right\}$ $= 846927.78 + 472477.92 - 349172.49$ $\boxed{I_{xx}\ 9.702 \times 10^5\,mm^4}$ $K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{9.702 \times 10^5}{3771.68}} \Rightarrow \boxed{K_{xx} = 16.04\,mm}$	**12**