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Department of Basic Science and Humanities

QUESTION BANK

ENGINEERING MATHEMATICS – II (17MAT21)-CBCS SCHEME

MODULE -1

DIFFERENTIAL EQUATIONS -1

Solve:

1. $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=0.\ given\ x(0)=0,\frac{dx}{xt}(0)=15.$
2. $\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=0.$
3. $(D^3+D^2+4D+4)y=0$
4. $(D^2-2D+4)^2y=0$
5. $(D^2+1)^3y$=0.
6. $\frac{d^4x}{dt^4}+4x=0.$
7. $\frac{d^3x}{dt^3}+y=0.$
8. $y''-2y'+10y=0, y(0)=4, y'(0)=1.$
9. $y'''+4y''+y'=0,$
10. $\frac{d^4y}{dt^4}+a^4y=0.$

Solve the following Differential Equations:

1. $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=(1-e^x)^2.$
2. $y''+4y'+4y=3sinx+4cosx, y(0)=1y'(0)=0.$
3. $(D-2)^2y$ =8$(e^{2x}+sin2x+x^2)$
4. $y''-2y'+2y=x+e^xcosx.$
5. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=xe^{3x}+sin2x$
6. $\frac{d^2y}{dx^2}-4y=xsinhx.$
7. $(D^2-1)y=xsin3x+cosx.$
8. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^xsinx.$
9. $(D^2-4D+4)y=8x^2e^{2x}sin2x.$
10. $\frac{d^2y}{dx^2}-a^2y=secax.$
11. $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=6e^{3x}+7e^{-2x}-log2.$
12. $\frac{d^3x}{dt^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+sin2x$

13. $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x.$
14. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + sinx + x.$
15. $(D^2 + a^2)y = tanax.$
16. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - cos^2x.$
17. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}.$
18. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}sin2x.$
19. $(D^3 + 2D^2 + D)y = x^2e^{-2x} + sin^2x.$
20. $(D^4 - 1)y = e^x cosx.$
21. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2coshx, Also\ find\ y\ when\ y = 0, \frac{dy}{dx} = 1\ at\ x = 0.$
22. $\frac{d^2x}{dt^2} + 2\frac{dx}{xt} + 3x = sint.$
23. $(D^2 - 4D + 3)y = sin3xcos2x$
24. $(D^3 - D)y = 2x + 1 + 4cosx + 2e^{x.}$
25. $\frac{d^4y}{dx^4} - y = cosxcoshx,$

Method of Variation of Parameters:

Solve:

1. $\frac{d^2y}{dx^2} + 4y = tan2x.$
2. $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}.$
3. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}.$
4. $\frac{d^2y}{dx^2} + a^2y = seeax.$
5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x sin2x.$
6. $\frac{d^2y}{dx^2} + y = 2cosx.$
7. $\frac{d^2y}{dx^2} + y = xsinx.$
8. $\frac{d^2y}{dx^2} + y = \frac{1}{1+sinx}$
9. $y'' - 6y' + 9y = \frac{e^x}{x^2}$
10. $y'' - 2y' + y = e^x logx.$

Method of Undetermined Coefficients

1. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=1-2x$
2. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$
3. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^x$
4. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{3x}+e^{2x}$
5. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=12x^2$
6. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=x^2+e^x$
7. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+4\frac{dy}{dx}=x^2+e^{-x}$
8. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{3x}+x$
9. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=x+\sin x$
10. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+y=2\cos x$
11. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+y=xe^x$
12. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}-y=x^2\cos x$
13. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+4y=xsi2x$

MODULE-2

Cauchy's Homogenous Linear equations:
Solve:

1. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=logx.$
2. $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}.$
3. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=logxsin(logx)$
4. $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+y=\frac{logsin(logx)+1}{x}.$
5. $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x.$

6. $\frac{d^2y}{dx^2} - \frac{2y}{x} = x + \frac{1}{x^2}$.
7. $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$
8. $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(logx)$
9. $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = xlogx$.
10. $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} = \frac{12logx}{x^2}$.

Legendre's Linear equations:

1. $(1+x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$.
2. $(2x-1)^2\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} - 2y = 8x^2 - 2x + 3$.c
3. $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$.
4. $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos(\log(1+x))$.
5. $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

Equations Solvable for p:

1. Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$
2. Solve: $p^2 + 2pycotx = y^2$
3. Solve: $y = x[p + \sqrt{1+p^2}]$
4. Solve: $p(p+y) = x(x+y)$
5. Solve: $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$

Equations Solvable for y:

1. Solve: $y - 2px = tan^{-1}(xp^2)$
2. Solve: $y = 2px + p^n$
3. Solve: $xp^2 + x = 2yp$
4. Solve: $y = psinp + cosp$
5. Solve: $y + px = x^4p^2$
6. Solve: $y = x + \alpha tan^{-1}p$

Equations Solvable for x:

1. Solve: $y = 2px + y^2p^3$
2. Solve: $p^3y + 2px = y$.
3. Solve: $x - yp = ap^2$
4. Solve: $p^3 - 4xyp + 8y^2 = 0$
5. Solve: $p = \tan\left(x - \frac{p}{1+p^2}\right)$

Clairaut's Equations:

1. Solve $p = \sin(y - xp)$ and also find the singular solution.
2. Solve: $(px - y)(py + x) = a^2p$.

3. Find the general solution and singular solution of
   (i) $xp^2 - yp + a = 0$
   (ii) $p = \log(p - xy$
4. Solve: $x^2(y - px) = yp^2$
5. Solve : $(px + y)^2 = py^2$
6. Solve: $(px + y)(x + py) = 2p$
7. Solve: $y + 2\left(\frac{dy}{dx}\right)^2 = (x + 1)\frac{dy}{dx}$

MODULE 3:

Form a PDE by eliminating arbitrary constants:

1. $Z= (x-a)^2+(y-b)^2$.
2. $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
4. $z = a\log\left\{\frac{b(y-1)}{1-x}\right\}$
5. Find the differential equation of all planes which are at a constant distance 'a' from the origin.

Form a PDE by eliminating arbitrary functions:

1. Z=f(y)+g(x+y)
2. Z= f(y-2x)+g(2y-x)
3. Z=f(x+at)+g(x-at)
4. Z= yf(x)+xg(y)
5. $Z=f(x^2-y^2)$
6. $f(x^2+y^2, z-xy)=0$.
7. $Z= y^2+2f(\{1/x\}+\log y)$.

Solving Homogenous PDE:

1. $\frac{\partial^2 z}{\partial y^2} - z = 0$ given that when y = 0 , z=e^x and $\frac{\partial z}{\partial y} = e^{-x}$.
2. $\frac{\partial^2 z}{\partial y^2} + z = 0$ given that when y = 0 , z=cosx and $\frac{\partial z}{\partial y} = \sin x$.

3. $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when x =0 , $\frac{\partial z}{\partial x} = a\sin y$ and $\frac{\partial z}{\partial y} = 0$.

4. $\frac{\partial^2 z}{\partial x^2} + 4z = 0$ given that when x =0 ,z=e^{2y} and $\frac{\partial z}{\partial x} = 2.$

Solving Non-Homogenous PDE:

1. $\frac{\partial^2 z}{\partial x^2 \partial y} = \sin(2x+3y)$.
2. $\frac{\partial^2 z}{\partial x^2 \partial y} = \cos(2x+3y)$.
3. $\frac{\partial^2 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x-y) = 0$.
4. $\frac{\partial^2 z}{\partial x^2 \partial y} = xy$.
5. $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x=0 and z=0 when y is an odd multiple of π/2.

Derive One dimensional heat equation and its solution by variable separable method.

Derive One dimensional wave equation and its solution by variable separable method.

MODULE 4:

Evaluate by changing the order of integration:

1. $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$
2. $\int_0^1 \int_0^{\sqrt{1-x2}} y^2\,dy\,dx$
3. $\int_0^{\infty} \int_0^x x.e^{-x^2/y}\,dy\,dx$

4. $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy$

Evaluate by changing into polar coordinates:

1. $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$
2. $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} e^{-(x^2+y^2)} dydx$

Evaluate the double or triple integration:

1. $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2) dzdydx.$
2. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdzdydx.$
3. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx.$
4. $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dzdydx^2$
5. $\int_0^1 \int_x^{\sqrt{x}} x^2+y^2 dydx.$
6. $\int_0^1 \int_x^{\sqrt{x}} xydydx.$
7. $\int_0^3 \int_1^{\sqrt{4-y}} x+ydydx$
8. $\int_1^2 \int_0^{2-y} xydydx.$

Finding Area and Volume:

1. Find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$
2. Show that area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $(16/3)a^2$.
3. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

4. Find the volume of the sphere $x^2+y^2+z^2=a^2$.
5. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
6. Find the area common to the cardioids r =a(1+cosΘ) and r=a(1-cosΘ).
7. Find the whole area of the cardioid r=a (1+cosΘ).
8. Find the whole area of lemniscate $r^2=a^2\cos 2\Theta$.

Beta -Gamma Functions:

1.Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

2.Prove that $\int_0^\infty x.e^{-x^8}dx.\int_0^\infty x^2e^{-x^4}dx=\frac{\pi}{16\sqrt{2}}$

3. Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx.\int_0^1 \frac{dx}{\sqrt{1+x^4}}=\frac{\pi}{4\sqrt{2}}$

4. Express in terms of beta and gamma functions and hence evaluate $\int_0^a x^4\sqrt{a^2-x^2}\,dx.$

5.Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$.

MODULE 5:

Find the Laplace transform of the following:

1.L{cos3t+2^t}.
2. L{e^{3t}sin3t.sin5t}
3. L{$e^{-t}\cos^2 3t$}
4.L{$e^{2t}\cos^2 t$}
5.L{t($\sin^3 t-\cos^3 t$)}
6.L{t^2e^{-3t}sin2t}
7.L{t^2e^{2t}}
8. $L\left\{\frac{1-\cos at}{t}\right\}$
9. $L\left\{\frac{\cos at-\cos bt}{t}+t\sin at\right\}$
10. $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$

11. $L\left\{\frac{\sin at}{t}\right\}$

12. $L\left\{2^t + \frac{\cos 2t - \cos 3t}{t}\right\}$

13.L{sint sin2t sin3t}

Find the Inverse Laplace transform of the following:

1. $L^{-1}\left\{\frac{3s+2}{s^2-s-2}\right\}$
2. $L^{-1}\left\{\frac{1}{s(s+1)(s+2)(s+3)}\right\}$
3. $L^{-1}\left\{\frac{s}{(2s-1)(3s-1)}\right\}$
4. $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$
5. $L^{-1}\left\{\frac{3s+2}{s^2-s-2}\right\}$
6. $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$
7. $L^{-1}\left\{\frac{s+1}{(s-1)^2(s+2)}\right\}$
8. $L^{-1}\left\{\frac{7s+4}{4s^2+4s+9}\right\}$
9. $L^{-1}\left\{\frac{s-2}{s^2+7s+12}\right\}$
10. $L^{-1}\left\{\frac{2s-1}{s^2+2s+17}\right\}$

Find the Laplace transform of the Periodic Function:

1.If f(t) is a periodic function T, Prove that $L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)dt$.

2.Given $f(t) = \begin{cases} E & a < t < a/2 \\ -E & a/2 < t < a \end{cases}$ where f(t+a) =f(t), then show that $L\{f(t)\} = \frac{E}{s}\tanh\left(\frac{as}{4}\right)$.

3. A Periodic function of period 2π/w is defined by $f(t)=\begin{cases} E\sin wt & 0\le t\le \pi/w \\ 0 & \pi/w\le t\le 2\pi/w \end{cases}$ where E and w are constants, Show that $L\{f(t)\}=\dfrac{Ew}{(s^2+w^2)(1-e^{-\pi s/w})}$

Find the Laplace transform of the Unit step function:

$$1.\ f(t)=\begin{cases} \sin t & 0\le t<\pi \\ \sin 2t & \pi\le t<2\pi \\ \sin 3t & t\ge 2\pi \end{cases}$$

$$2.\ f(t)=\begin{cases} t^2 & 0<t<2 \\ 4t & 2<t<4 \\ 8 & t>4 \end{cases}$$

$$3.\ f(t)=\begin{cases} 1 & 0<t\le 1 \\ t & 1<t\le 2 \\ t^2 & t>2 \end{cases}$$

$$4.\ f(t)=\begin{cases} \cos t & 0<t<\pi \\ \cos 2t & \pi\le t<2\pi \\ \cos 3t & t>2\pi \end{cases}$$

$$5.\ f(t)=\begin{cases} \cos t & 0<t<\pi \\ \sin t & t>\pi \end{cases}$$

$$6.\ f(t)=\begin{cases} t^2 & 1<t\le 2 \\ 4t & t>2 \end{cases}$$

Find the Inverse Laplace transform of the following using Convolution:

1. $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

2. $L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$

3. $L^{-1}\left\{\frac{s}{(s+2)(s^2+9)}\right\}$

4. $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$

5. $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$

Employ Laplace transform to solve the ODE:

1. $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{-x}; \quad y(0)=0, y'(0)=0.$

2. $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t; \quad y(0)=0, y'(0)=0.$

3. $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=5e^{2x}; \quad y(0)=2, y'(0)=1.$

4. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=1-e^{2x}; \quad y(0)=1, y'(0)=1.$

5. $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4t+e^{3t}; \quad y(0)=1, y'(0)=-1.$

6. $\frac{d^2y}{dt^2}+6\frac{dy}{dt}+9y=12t^2e^{-3t}; \quad y(0)=0, y'(0)=0.$

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