PESIT Bangalore South Campus
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Department of Electronics & Communication Engineering

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Internal Assesment Test - 2

Date: 6 March. 2017
Subject & Code: **Det. and Est.** 16*ESP*251
Name of faculty: Bharath B. N.

Marks: 50
Section: M-tech SP
Time: 8 : 30AM to 10 : 00AM

1 Questions (Answer any 5 out of 7 questions)

1. Show that the conditional expectation is the optimal estimator with respect to the mean square error as a cost function. (10 **points)**

2. Suppose the observation model is given by

$$\mathbf{r} = a + \mathbf{n}, \tag{1}$$

where $a \in \mathbb{R}$ is a fixed but unknown quantity, and $\mathbf{n} \sim \mathcal{N}(0, \sigma_n^2)$. Find the MMSE estimate of a. (10 **points)**

3. Consider the following hypothesis testing problem

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \sim \begin{cases} \mathcal{N}(0, \sigma^2 I_{N\times N}) & \text{under } \mathcal{H}_0 \\ \mathcal{N}(\mathbf{m}, \sigma^2 I_{N\times N}) & \text{under } \mathcal{H}_1. \end{cases} \tag{2}$$

where $\mathbf{m} \in \mathbb{R}^{N\times 1}$ is the mean vector. Find the optimal decision rule for this problem. (10 **points)**

4. List all the properties of an ML estimator. (10 **points)**

5. Find the optimal estimate with respect to the following cost functions: (i) absolute error and (ii) $C_\tau(\hat{a}) = 1$ if $|\hat{a}| > \tau$, and zero otherwise for some $\tau > 0$. (10 **points)**

6. Define an unbiased estimator, and derive the Cramer-Rao Lower Bound (CRLB) for any unbiased estimator. (10 **points)**

7. Find an ML estimate of the mean of a poisson process given N observations, i.e.,

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \overset{(i.i.d)}{\sim} \texttt{Poiss}(\lambda) \tag{3}$$

In the above, $\texttt{Poiss}(\lambda)$ denotes the Poisson distribution with mean λ given by

$$\Pr\{X = k\} = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, \ldots, . \tag{4}$$

Verify if the above ML estimator achieves the CRLB. *Hint:* Find the mean square error in the ML estimate, and check if it is equal to the CRLB. (10 **points)**

2 Solution

- See class notes for all the solutions.