PESIT Bangalore South Campus
Hosur road, 1km before Electronic City, Bengaluru -100
Department of Electronics & Communication Engineering

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Internal Assesment Test - 1

Date: 1 March. 2017
Subject & Code: **Det. and Est. 16ESP251**
Name of faculty: Bharath B. N.

Marks: 50
Section: M-tech SP
Time: 8:30AM to 10:00AM

1 Questions (Answer any 5 out of 7 questions)

1. Show that the Likelihood Ration (LR) test is optimal in the Bayesian sense for optimally detecting the binary hypothesis. **(10 points)**
   Solution: See text book.

2. Explain the Neyman-Pearson test, and show that the LR is optimal in the Neyman-Pearson sense. **(10 points)**
   Solution: See text book.

3. Consider the following hypothesis testing problem

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \overset{(i.i.d)}{\sim} \begin{cases} \mathcal{N}(0,1) & \text{under } \mathcal{H}_0 \\ \mathcal{N}(a,1) & \text{under } \mathcal{H}_1. \end{cases} \tag{1}$$

   Design an optimal Neyman-Pearson test for the false alarm level of α.
   Solution: See text book.

4. Consider the following hypothesis testing problem

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \overset{(i.i.d)}{\sim} \begin{cases} \texttt{uniform}(0,1) & \text{under } \mathcal{H}_0 \\ \texttt{uniform}(0,2) & \text{under } \mathcal{H}_1. \end{cases} \tag{2}$$

   Design an optimal Neyman-Pearson test for the false alarm level of α. Here, $\texttt{uniform}(0, a)$ denotes a uniform distribution in the interval $(0, a)$.
   Solution: The likelihood ratio test is

$$\max\{R_1, R_2, \ldots, R_N\} \overset{>}{<} \eta. \tag{3}$$

   Find η such that the false alarm is satisfied. See notes for the details.

5. Define the ROC, and prove at least three of its properties.
   Solution: See text book.

6. Consider the following hypothesis testing problem

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \overset{(i.i.d)}{\sim} \begin{cases} \texttt{uniform}(0,1) & \text{under } \mathcal{H}_0 \\ \texttt{uniform}(0,2) & \text{under } \mathcal{H}_1. \end{cases} \tag{4}$$

   Design an optimal Bayesian test when the prior $P_0 = 0.8$, and the costs $C_{00} = C_{11} = 0$, and $C_{10} = C_{01} = 1$.
   Solution: Similar to the above.

7. Design an optimal LR rule for the following observation model:

$$(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \overset{(i.i.d)}{\sim} \begin{cases} \texttt{Poiss}(1) & \text{under } \mathcal{H}_0 \\ \texttt{Poiss}(5) & \text{under } \mathcal{H}_1. \end{cases} \tag{5}$$

In the above, $\texttt{Poiss}(\lambda)$ denotes the Poisson distribution with mean λ given by

$$\Pr\{X = k\} = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, \ldots, . \tag{6}$$

(10 **points)**
Solution: See the class notes.